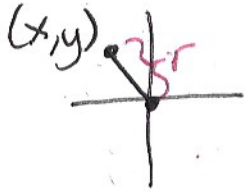
TO CONVERT (x,y) TO (r,θ)

$x = r\cos\theta$

$y = r\sin\theta$



$r = \sqrt{x^2+y^2}$ $\quad r^2 = x^2+y^2$

$\cancel{\theta = \underbrace{\tan^{-1}\frac{y}{x}}_{\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)}}$ $\quad \tan\theta = \frac{y}{x}$

$$\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta} = \tan\theta$$

eg. CONVERT $= \underset{}{\overset{(x,y)}{\left(-3\sqrt{3},9\right)}}$ TO POLAR

$$r^2 = x^2+y^2 = \left(-3\sqrt{3}\right)^2 + 9^2$$

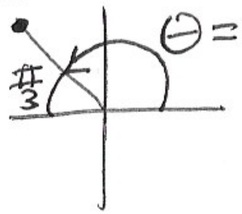
$$= 27+81 = 108$$

$$r = \sqrt{108} = 6\sqrt{3}$$

$$\tan\theta = \frac{y}{x} = \frac{9}{-3\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = -\sqrt{3}$$

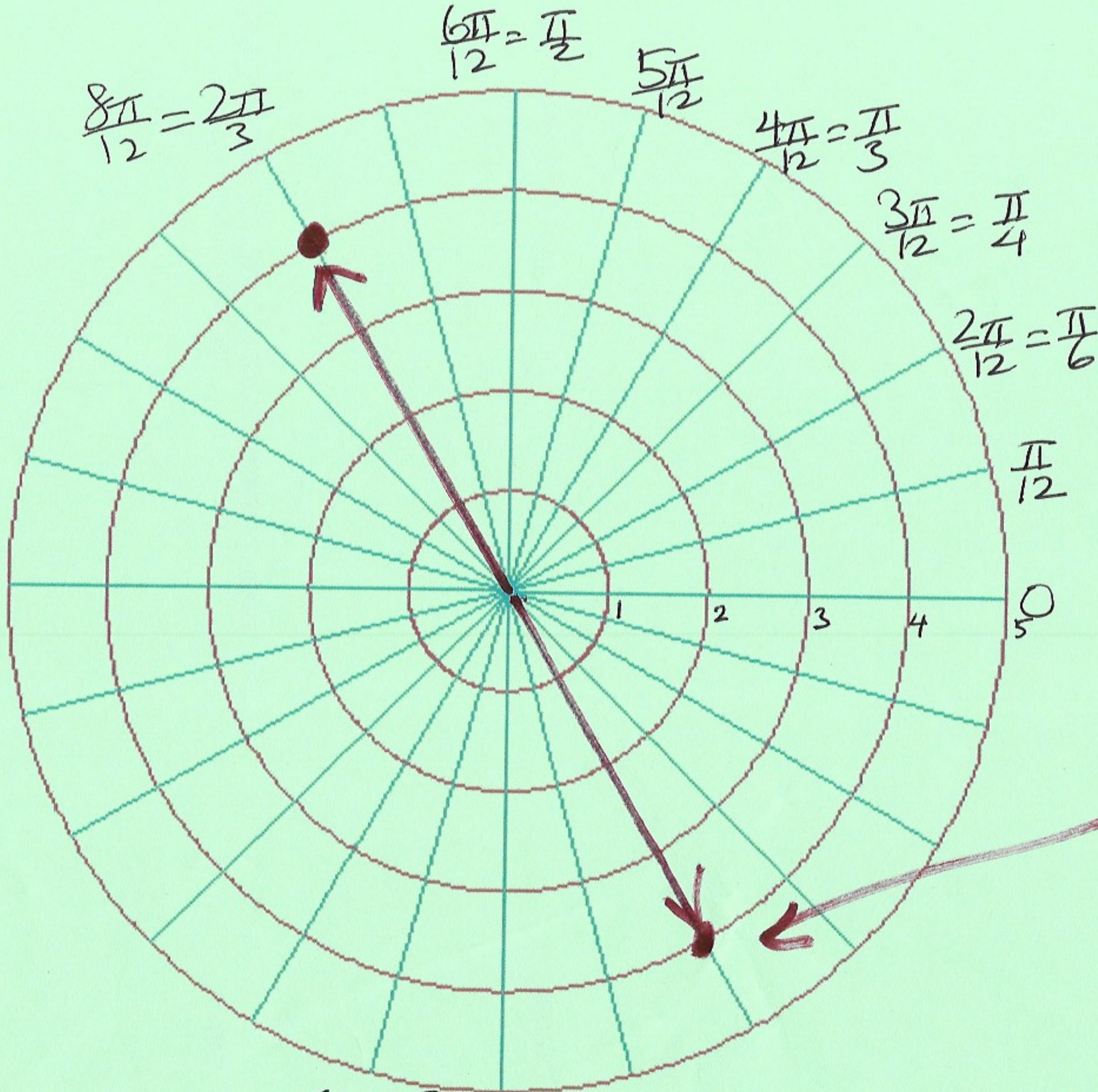
$\uparrow$

$\theta_{REF} = \frac{\pi}{3}$



$\theta = \frac{2\pi}{3} = \pi - \frac{\pi}{3}$

$$(r,\theta) = \left(6\sqrt{3}, \frac{2\pi}{3}\right)$$



PLOT $\underline{\left(4, \frac{2\pi}{3}\right)}$

A/K/A

$\left(4, \frac{2\pi}{3} + 2n\pi\right)$

$n \in \mathbb{Z}$

$x = 4\cos\frac{2\pi}{3}$

$y = 4\sin\frac{2\pi}{3}$

WHAT IS

$\left(-4, \frac{2\pi}{3}\right)$?

$x = -4\cos\frac{2\pi}{3}$

$y = -4\sin\frac{2\pi}{3}$

$(-r, \theta)$ IS THE SAME POINT AS $(r, \theta + \pi)$

(r, θ) IS THE SAME POINT AS $(-r, \theta + \pi)$

(r, θ) IS THE SAME POINT AS $(r, \theta + 2n\pi)$

AND $(-r, \theta + \pi + 2n\pi)$

CONVERT $y = x^2$ TO POLAR $\leftarrow$ $r = f(\theta)$

OR

$r^2 = f(\theta)$

$\vdots$

SUBSTITUTE $x = r\cos\theta$

$y = r\sin\theta$

$$r\sin\theta = (r\cos\theta)^2$$

$$r\sin\theta = r^2\cos^2\theta$$

$$0 = r^2\cos^2\theta - r\sin\theta$$

$$0 = r(r\cos^2\theta - \sin\theta)$$

$r = 0$ OR $r\cos^2\theta - \sin\theta = 0$

$$r = \frac{\sin\theta}{\cos^2\theta} = \sec\theta\tan\theta$$

$r = \sec 0 \tan 0 = 0$

IE. $r = 0$ IS ALREADY

PART OF $\boxed{r = \sec\theta\tan\theta}$

CONVERT $r = \dfrac{1}{1+\sin 2\theta}$ TO RECTANGULAR

(*) ELIMINATE FRACTIONS IMMEDIATELY

(1) USE TRIG IDENTITIES SO THAT THE ONLY ANGLE INSIDE A TRIG FUNCTION IS θ

(IE. NO 2θ, $\frac{1}{2}\theta$, $\theta+\frac{\pi}{2}$)

(2) SUBSTITUTE

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

$\csc\theta = \frac{r}{y}$ $\sec\theta = \frac{r}{x}$ $\cot\theta = \frac{x}{y}$

(3) SUBSTITUTE $r^2 = x^2+y^2$ OR $r = \sqrt{x^2+y^2}$

+ SIMPLIFY

(*) $r(1+\sin 2\theta) = 1$

(1) $r(1+2\sin\theta\cos\theta) = 1$

(2) $r(1+2\frac{y}{r}\frac{x}{r}) = 1$

(*) $r + \frac{2xy}{r} = 1$

$r^2 + 2xy = r$

$\rightarrow x^2+y^2+2xy = \sqrt{x^2+y^2}$

$((x+y)^2)^2 = x^2+y^2$

$(x+y)^4 = x^2+y^2$

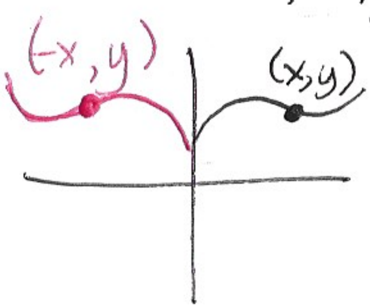
GRAPH $r = \sin\theta$

$\sqrt{2} \approx 1.4$

$\sqrt{3} \approx 1.8$

θ	r	(r, θ)
0	$\sin 0 = 0$	$(0, 0)$
$\frac{\pi}{6}$	$\sin\frac{\pi}{6} = 0.5$	$(0.5, \frac{\pi}{6})$
$\frac{\pi}{4}$	$\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \approx 0.7$	$\approx (0.7, \frac{\pi}{4})$
$\frac{\pi}{3}$	$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \approx 0.9$	$\approx (0.9, \frac{\pi}{3})$
$\frac{\pi}{2}$	$\sin\frac{\pi}{2} = 1$	$(1, \frac{\pi}{2})$

TO TEST IF $f(x,y) = C$ IS SYM OVER y-AXIS



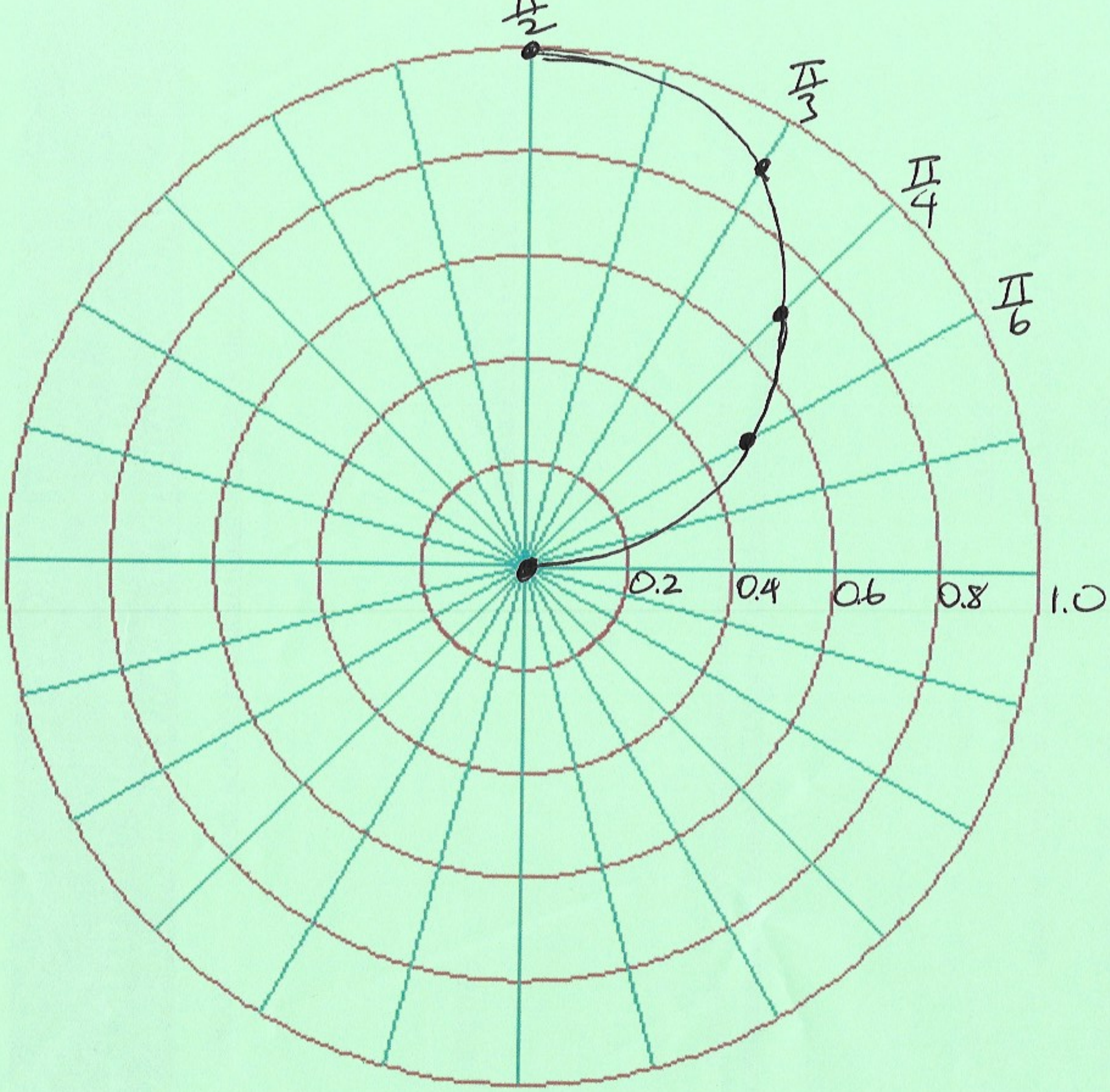
REPLACE (x,y) WITH $(-x,y)$

DOES NOT SIMPLIFY

IF THE NEW EQ'N SIMPLIFIES BACK TO THE ORIGINAL EQ'N,

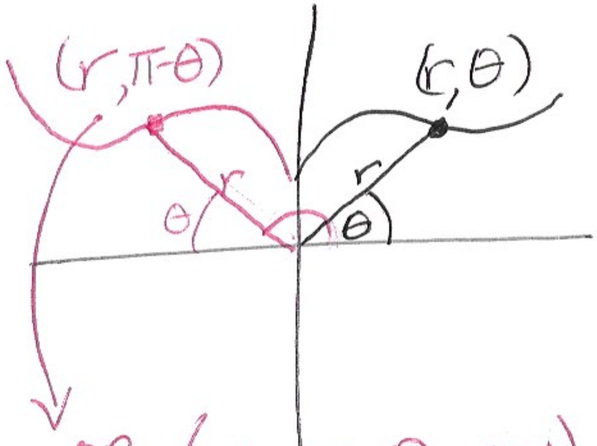
NOT SYM

THEN GRAPH IS SYM OVER y-AXIS



π/2
π/3
π/4
π/6
0.2
0.4
0.6
0.8
1.0

TO TEST IF $f(r,\theta)=C$ IS SYM OVER $\theta=\frac{\pi}{2}$



OR $(r, \pi-\theta+2\pi)$

$(r, \pi-\theta+4\pi)+\cdots$

OR $(-r,-\theta)$

$(-r,-\theta+2\pi)$

$(-r,-\theta+4\pi)\ldots$